

# Opaque or transparent? A link between neutrino optical depths and the characteristic duration of short Gamma Ray Bursts

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## ABSTRACT

Cosmological gamma ray bursts (GRBs) are thought to occur from violent hypercritical accretion onto stellar mass black holes, either following core collapse in massive stars or compact binary mergers. This dichotomy may be reflected in the two classes of bursts having different durations. Dynamical calculations of the evolution of these systems are essential if one is to establish characteristic, relevant timescales. We show here for the first time the result of dynamical simulations, lasting approximately one second, of post-merger accretion disks around black holes, using a realistic equation of state and considering neutrino emission processes. We find that the inclusion of neutrino optical depth effects produces important qualitative temporal and spatial transitions in the evolution and structure of the disk, which may directly reflect upon the duration and variability of short GRBs.

*Subject headings:* accretion, accretion disks — gamma rays: bursts — dense matter — hydrodynamics — neutrinos

## 1. Introduction

The coalescence of two compact objects due to the emission of gravitational radiation is a possible origin of cosmological gamma ray bursts of the short variety (see e.g., Mészáros 2002, for a review), lasting a few tenths of a second (Lattimer & Schramm 1976; Paczyński 1986; Eichler et al. 1989; Narayan, Paczyński & Piran 1992). Previous multidimensional studies have shown that the outcome of such a merger is in all likelihood a dense torus surrounding a supramassive neu-

tron star, likely to collapse to a black hole on a short timescale, or a black hole, if one was already present in the system (e.g. Rasio & Shapiro 1994; Kluźniak & Lee 1998; Ruffert & Janka 1999; Ross-wog, Ramirez-Ruiz & Davies 2003). This class of systems has been studied observationally since the 1970s, when the Hulse–Taylor system was discovered (Hulse & Taylor 1975), and the recent detection of PSR J0737-3039 (Burgay et al. 2003), the tightest binary yet in this class, has only heightened interest in these events. The final instants before the actual merger will produce a powerful burst of gravitational waves, and the merger waveform itself is expected to reveal details about the

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equation of state of matter at high densities (see e.g., Thorne 1995).

The merger process is intrinsically a three-dimensional event, with no particular symmetry which can be exploited to reduce the complexity of the problem, and must be addressed numerically. No such calculation is currently evolved for more than a few tens of milliseconds, because of computational limitations. Information about the state of the system after the violent, dynamical merger is essentially over can be obtained, but the final state of the system, on a timescale comparable to the duration of a short GRB, cannot be addressed.

The steady state structure and composition of post-merger accretion disks has been the focus of several recent studies (Popham, Woosley & Fryer 1999; Narayan, Piran & Kumar 2001; Kohri & Mineshige 2002; Di Matteo, Perna & Narayan 2002; Pruet, Woosley & Hoffman 2003; Beloborodov 2003), which have included an increasing amount of microphysical detail. However, the dynamics of the problem have never been followed for an extended amount of time, although progress is being made in this direction (Setiawan, Ruffert & Janka 2004).

In a previous paper (Lee & Ramirez-Ruiz 2002), we studied the dynamical evolution of the accretion structures formed after a black hole–neutron star merger under a set of simplifying assumptions. Initial conditions were taken directly from the late stages of the coalescence, modeled in three dimensions using a simple equation of state (Lee 2001). The disk was then mapped onto two dimensions assuming azimuthal symmetry, and its evolution was followed for 0.2 seconds, comparable to the duration of a short GRB. The equation of state was simple (ideal gas with adiabatic index  $\gamma = 4/3$ ), and we assumed that all the energy dissipated by viscosity (modeled with the full expression of the stress tensor, using an  $\alpha$ -prescription for the magnitude of the viscosity) was radiated away.

In this *Letter* we present for the first time dynamical calculations of post-merger accretion disks in two dimensions,  $(r, z)$ , that: (1) use a realistic and self-consistent equation of state for the fluid in the disk; (2) consider the cooling of the disk through neutrino emission, using appropriate rates; (3) account for the effects of neutrino opacities

and the corresponding optical depths through a simplified treatment and (4) last for a minimum of 0.4 seconds, comparable to the duration of a short GRB.

## 2. Physics

After the initial merger phase of the binary is completed, a few tenths of a solar mass can be left in orbit around the central black hole (which has a mass  $M_{\text{BH}} \approx 3\text{--}5 M_{\odot}$ ), in a small disk a few hundred kilometers across. The densities are high, with  $10^9 \leq \rho (\text{g cm}^{-3}) \leq 10^{12}$ , and the corresponding temperatures are  $10^{10} \leq T(\text{K}) \leq 10^{11}$ . Under these conditions, nuclei are entirely photodisintegrated, and the fluid consists of free electrons, protons, nucleons, plus a small number of positrons. The nucleons make up a non-relativistic ideal gas, while electrons are degenerate and highly relativistic. Photons are completely trapped due to the extremely high optical depth, and are hence advected with the flow. The composition of the fluid is determined self-consistently assuming: (1) charge neutrality and (2) complete  $\beta$ -equilibrium (Kohri & Mineshige 2002; Lee, Ramirez-Ruiz & Page 2004). This allows us to determine the electron fraction  $Y_e$  and the electron degeneracy parameter (through the chemical potential) locally, and to follow it throughout the dynamical evolution. This is important since the composition directly affects the neutrino emission rates (see below). The equation of state is then given by

$$P = \frac{aT^4}{3} + \frac{\rho kT}{m_u} + K \left( \frac{\rho}{\mu_e} \right)^{4/3} + P_{\nu}, \quad (1)$$

where  $K$  is the constant corresponding to a fully degenerate, relativistic electron gas,  $\mu_e = 1/Y_e$  is the number of baryons per electron, and  $P_{\nu}$  is the pressure due to the presence of neutrinos in the optically thick regions of the disk (see below). Strictly speaking, the electrons are not fully degenerate, since  $\eta_e/kT$  is in the range 2–4 ( $\eta_e$  is the electron chemical potential). However, this represents a small error in the equation of state, since gas pressure dominates the overall balance, contributing about 90% of the total pressure, followed by electron degeneracy pressure at around 5%.

Neutrinos are emitted abundantly in the disk, due to the high temperatures and densities. We

take into account all of the following processes: (1) nucleon–nucleon bremsstrahlung; (2)  $e^\pm$  pair annihilation; (3) electron and positron captures onto free nucleons and (4) plasmon decay. In practice, the global cooling rate is completely dominated by  $e^\pm$  captures, and is hence well approximated by (Kohri & Mineshige 2002):

$$\dot{q}_{cap} = 1.1 \times 10^{31} \left[ \frac{\eta_e}{kT} \right]^9 T_{11}^9 \text{ erg s}^{-1} \text{ cm}^{-3} \quad (2)$$

in the regime where electrons are degenerate. This expression has been used directly to compute the cooling within the disk. The error in doing so, as opposed to a full integration over the phase space, is limited to about a factor of 2 (in the inner regions, where the cooling is most intense), as compared with rates extrapolated from the work of Langanke & Martínez-Pinedo (2001).

The emitted neutrinos are not entirely free to leave the system, since scattering off free nucleons is important. The optical depth for neutrinos can be estimated as  $H/l_\nu$ , where  $H$  is a typical disk scale height, and  $l_\nu$  is the mean free path between scatterings. The cross section depends on the mean neutrino energy as  $\sigma_n \propto E_\nu^2$  (Tubbs & Schramm 1975), and  $E_\nu \approx 43(Y_e \rho_{12})^{1/3} \text{ MeV}$  ( $\rho_{12} = \rho/10^{12} \text{ g cm}^{-3}$ ), using the fact that neutrinos are being produced by reactions involving degenerate electrons (see e.g., Shapiro & Teukolsky 1983). This amounts to a surface of last scattering for neutrinos, or “neutrino–surface” at  $\rho \simeq 10^{11} \text{ g cm}^{-3}$ . Our simple treatment of this fact consists of suppressing the local cooling rate by a factor  $\exp(-\tau_\nu)$ , and adding a pressure term  $P_\nu \propto aT^4[1 - \exp(-\tau_\nu)]$  to the equation of state. This simple alteration is crucial in determining the energy output of the disk. We note that no modification in the composition was computed owing to this fact. In reality, the opaqueness of the material will lead to an increase in the electron fraction  $Y_e$ , by up to a factor of 2 (Beloborodov 2003). The optical depth will thus rise by a factor  $2^{2/3} \simeq 1.6$ , making the optically thick region in the disk larger and the overall luminosity slightly lower. Any energy release in neutrinos will thus be spread over a longer period, making our estimates concerning time scales fall on the conservative side (see below, Section 3).

The dynamical evolution is followed for at least 0.4 seconds in the central Newtonian potential

produced by the black hole,  $\Phi = -GM_{\text{BH}}/R$ , using a two dimensional smooth particle hydrodynamics (SPH) code in azimuthal symmetry (Monaghan 1992). Accretion is modeled by placing an absorbing boundary at the Schwarzschild radius  $r_G = 2GM_{\text{BH}}/c^2$ , and the mass of the black hole is updated continuously. There is no external agent feeding the disk with matter, and no boundary conditions need to be specified (other than for accretion). The equations of motion and the energy equation contain all the terms for a physical viscosity derived from the stress tensor (Flebbe et al. 1994), and we use an  $\alpha$ -prescription for the coefficient of viscosity, with  $10^{-3} < \alpha < 10^{-1}$ . The dissipated energy is injected back into the disk as thermal energy, and may leave the system depending on the local cooling rate (see above).

### 3. Temporal and spatial transitions

If the disk is assumed to be optically thin everywhere, and the effects discussed above are ignored, the neutrino luminosity is a smooth, monotonically decreasing function of time, similar to what we obtained in previous work (we have confirmed this in trial runs with the present input physics). Accounting for a variable optical depth leads to behavior that is markedly different, as can be seen in Figure 1, where  $L_\nu(t)$  is shown for three values of the viscosity parameter  $\alpha$ . For a high viscosity,  $\alpha = 0.1$ , the accretion timescale is so short (40 ms), that most of the material is accreted onto the black hole and has no time to radiate away its internal energy reservoir of  $\approx 10^{52} \text{ erg}$ . The disk becomes entirely optically thin at around 30 ms, and this explains the rebrightening that is observed. The luminosity subsequently exhibits a power law decay, as in the trial runs with no optical depth effects, with  $L_\nu \propto t^{-0.9}$ .

For lower viscosities ( $\alpha = 10^{-2}, 10^{-3}$ ), the behavior is qualitatively different at early times, and is a consequence of the longer accretion timescales: 0.4 s and 1 s respectively (we note here that all our estimates of  $t_{\text{acc}}$  are based on the actual accretion rate, i.e.  $t_{\text{acc}} \approx M_d/\dot{M}_{\text{BH}}$ ). Quite simply the material in the disk is allowed to remain in the vicinity of the black hole without being accreted, at high temperatures and densities (since the transport of angular momentum is so inefficient), and radiate away essentially all of its in-

ternal energy through the emission of neutrinos. Once this occurs, on a cooling timescale given by  $t_{\text{cool}} \approx E_{\text{int}}/L_{\nu}$ , a break occurs (at  $t \simeq 80, 100$  ms for  $\alpha = 10^{-2}, 10^{-3}$  respectively) and the luminosity drops again as  $t^{-0.9}$ . At this stage the energy output comes directly from viscous dissipation within the disk. The break time is determined by how much energy the disk holds initially,  $E_{\text{int}}$ , and by the rate at which it is lost,  $L_{\nu}(t)$ , which is itself fixed by the densities and temperatures found within it. The limitation on the cooling rate imposed by high optical depths is essential, and allows the energy loss to be spread over an extended period of time. For the three calculations shown here, the total energy release in neutrinos, up to 0.4 s, is  $(E_{\nu}/10^{52} \text{ erg}) = 1.2, 1.05, 0.8$ , for  $\alpha = 10^{-1}, 10^{-2}, 10^{-3}$ , respectively. Assuming that this energy is converted to  $e^{\pm}$  with a one per cent efficiency in the regions along the rotation axis through  $\nu\bar{\nu}$  annihilation, approximately  $10^{50}$  erg could be released and made available for the production of a relativistic fireball. A detailed calculation of this is left as future work.

The other important mechanism by which a GRB might be powered is MHD energy extraction, either through the Blandford–Znajek mechanism (Blandford & Znajek 1977), or through a magnetically dominated jet. Since our code does not explicitly account for the presence of magnetic fields, we make a simple, rough estimate of the magnetic field strength by assuming that its energy density,  $B^2/8\pi$ , is in equipartition with the internal energy density,  $\rho c_s^2$  (this is clearly an upper limit). Within the disk, the field will be most intense where the density is highest. Figure 2 shows the maximum density in the disk as a function of time, for the same calculations as shown in Figure 1. The inferred magnetic field is  $B \simeq 10^{15} - 10^{16}$  G. It is again evident that the high-viscosity case is markedly different from the other two, showing a significant prompt decrease in  $\rho_{\text{max}}$ . The transition to complete transparency to neutrinos occurs at around 40 ms. The balance between cooling, which produces vertical compression (and hence a rise in central density), and accretion, which slowly reduces the mass of the disk, produces a break in the curves for  $\alpha = 0.01, 0.001$  at  $t \approx 0.05, 0.8$  s respectively. The total estimated energy release through the Blandford–Znajek mechanism is  $(E_{\text{BZ}}/10^{51} \text{ erg}) = .34, 1.3, 6.5$ , for  $\alpha = 10^{-1}$  (up

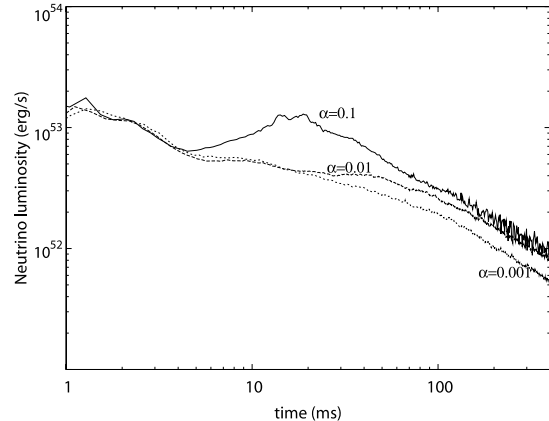


Fig. 1.— Neutrino luminosity as a function of time for three runs using identical initial conditions and different values of the viscosity parameter:  $\alpha = 10^{-1}$ ,  $\alpha = 10^{-2}$  and  $\alpha = 10^{-3}$ .

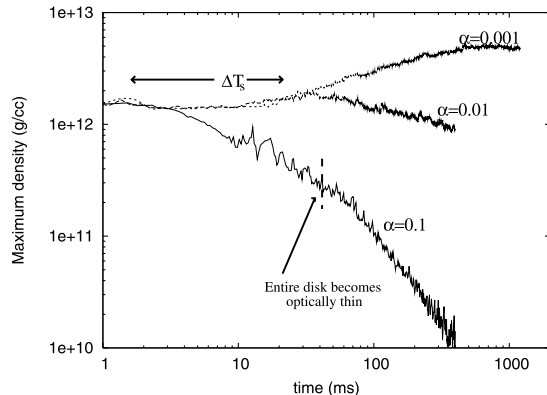


Fig. 2.— Maximum density as a function of time for the same runs as shown in Figure 1. The interval marked  $\Delta T_s$  corresponds to the sound crossing time across the optically thick region of the disk. The subsequent rise is due to the vertical compression of the disk as it cools. Accretion eventually drains the disk of matter, producing the characteristic breaks at  $t = 0.05, 0.8$  s for  $\alpha = 10^{-2}, 10^{-3}$ .

to 0.4 s),  $\alpha = 10^{-2}$  (up to 0.4 s),  $\alpha = 10^{-3}$  (up to 1.2 s), respectively.

In addition to the temporal variations in total luminosity, there is a spatial transition within the disk, when the optical depth is around unity. For the calculations with  $\alpha \leq 10^{-2}$ , there is always (up to the end of the run) an opaque region in the center of the disk. The density  $\rho$  and entropy per baryon  $s/k$  along the equatorial plane ( $z = 0$ ), as well as density contours in the  $r - z$  plane, are plotted in Figure 3, 0.2 s after the beginning of the calculation with  $\alpha = 10^{-2}$ . We also show for comparison the equatorial density profile of the disk in a calculation that did not take into account the effects of optical depths (dashed line in Figure 3). There is a clear break at  $r \approx 9 \times 10^6$  cm, where  $\tau_\nu = 1$ . As the optical depth increases and the cooling is suppressed, the pressure rises, inhibiting the thinning of the disk and the increase in density in the inner regions. This break is essentially a stationary feature, due to the relatively long accretion time scales, as long as the optically thick region is present. We believe this transition is significant, as geometrically thick accretion flows around rotating black holes, with  $H \simeq R$ , are believed to be more favorable for the production of energetic outflows than thin flows, where  $H \ll R$  (Livio, Ogilvie & Pringle 1999; Meier 2001).

#### 4. Discussion

The evolution of accretion disks resulting from dynamical three dimensional binary coalescence calculations, where a neutron star is tidally disrupted before being swallowed by its black hole companion, is studied numerically. Angular momentum transport and the associated energy dissipation are modeled using an  $\alpha$  prescription. By assuming azimuthal symmetry we are able to follow the time dependence of the disk structure for a fraction of a second, a time comparable with the duration of short GRBs. During this time, the rate of mass supply to the central black hole is of the order of a fraction of a solar mass per second; i.e. much greater than the Eddington rate. Although the gas photon opacities are large, the disk becomes sufficiently dense and hot to cool via neutrino emission. There is in principle no difficulty in dissipating the disk internal energy, but the problem is in allowing these neutrinos to es-

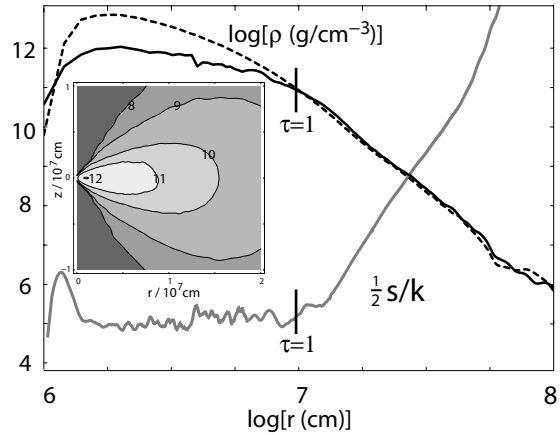


Fig. 3.— Density profile and entropy per baryon along the equatorial plane,  $z = 0$ , 0.2 s after the start of the calculation with  $\alpha = 10^{-2}$  (solid line). The dashed line indicates the density profile for the corresponding trial run that did not include optical depth effects. The vertical mark on each curve indicates the position where  $\tau_\nu = 1$ . *Inset Panel:* Logarithmic density contours (equally spaced every dex). The neutrino-surface corresponds roughly to  $\log \rho = 10^{11}$  g cm $^{-3}$ . Under these circumstances, the gas cannot efficiently cool and maintains a pressure scale height comparable with its radius. This, however, cannot happen too close to the symmetry axis where the centrifugal force cannot oppose the vertical pull of gravity and a funnel is formed.

cape from the Thomson thick inflowing gas.

At sufficiently low accretion rates,  $\alpha \lesssim 0.01$ , we find that the energy released by viscous dissipation is almost completely radiated away on a timescale given by  $t_{\text{cool}} \approx E_{\text{int}}/L_\nu \sim 0.1$  s. In contrast, for a higher mass supply,  $\alpha \gtrsim 0.1$ , energy advection remains important until the entire disk becomes optically thin, at  $t[\tau_\nu = 1] \sim 30$  ms. The restriction on the cooling rate imposed by high optical depths is key because it allows the energy loss to be spread over an extended period of time (i.e.  $t_{\text{cool}}$  or  $t[\tau_\nu = 1]$ ) during which the neutrino luminosity stays roughly constant. In principle, neutrinos could give rise to a relativistic pair dominated wind if they converted into  $e^\pm$  pairs in a region of low baryon density. This gives a characteristic timescale for energy extraction, and may be

essential for determining the duration of neutrino-driven, short GRBs.

An alternative way to tap the torus energy is via magnetic fields threading the disk: the energy liberated by accretion is converted efficiently into magnetic form and emitted as a magnetically dominated outflow. The launching of a jet probably requires the existence of a poloidal magnetic field of magnitude  $B_p$  over a scale of radius  $R$ , where  $B_p$  is smaller than the field strength  $B_{\text{disk}}$  associated with the dynamo-driven magnetic disk viscosity (Livio, Ogilvie & Pringle 1999). This results directly from estimating the accretion and jet luminosities as  $L_{\text{acc}} \sim \dot{M}_{\text{acc}} v_\phi^2$ , and  $L_{\text{jet}} \sim B_p^2 R^2 v_\phi$  respectively, which gives  $(B_p/B_{\text{disk}})^2 \sim (L_{\text{jet}}/L_{\text{acc}})(H/R)$ . Assuming the scaling  $B_p \sim (H/R)B_{\text{disk}}$ , derived by Tout & Pringle (1996), one obtains  $L_{\text{jet}}/L_{\text{acc}} \sim H/R$ . In order to achieve  $L_{\text{jet}} \sim L_{\text{acc}}$ , Livio, Pringle & King (2003) argue that a number  $R/H$  of neighboring annuli in the disk need to provide a locally generated net poloidal field in the same direction. They notice that since the local dynamos vary on timescales of  $t_{\text{dyn}}$ , the timescale for establishing a change in the magnetic field in the disk should be of the order  $t_{\text{jet}} \sim t_{\text{dyn}} 2^{R/H}$ . In our case  $R/H \sim 3$ , which gives  $t_{\text{jet}} \sim 10 - 100$  ms. This corresponds to the typical variability timescale of short duration GRBs (Ramirez-Ruiz & Fenimore 2000; Nakar & Piran 2002).

Two further physical effects, noted previously in a more general context, are relevant for the present study. First, the mechanism by which a jet is launched is not scale-free (Pringle 1993), and thus the poloidal field must be established in the inner regions of the disk. Second, in these same regions, the inflow speed caused by angular momentum loss to the outflow is increased, up to a factor  $R/H$  over the value expected from magnetic viscosity alone (Livio, Pringle & King 2003). Thus the poloidal flux generated by the dynamo can be effectively trapped. This probably implies that once the dynamo process generates a global poloidal field, it is able to maintain it during the lifetime of the thick portion of the disk,  $t[\tau_\nu = 1]$ . For the typical parameters studied here, this timescale could be of the order of seconds when the rate of mass supply is low (i.e.  $\alpha \sim 10^{-3}$ ). There is thus no problem in principle in accounting for sporadic large-amplitude vari-

ability on timescales as short as one millisecond, even in the most long-lived short GRBs.

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